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Reasonable Design Space Approach to Response Surface Approximation

Vladimir O. Balabanov,* Antony A. Giunta,* Oleg Golovidov,* Bernard Grossman,†
William H. Mason,‡ and Layne T. Watson§
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
and
Raphael T. Haftka¶
University of Florida, Gainesville, Florida 32611-6250

Response surface approximations have become very popular as a tool for multidisciplinary optimization. However, the accuracy of these models is a major concern. One method to increase the accuracy of response surface approximations is to construct them only for the portion of the design space that represents reasonable designs. Our paper describes the methodology underlying this reasonable design space approach. This methodology is illustrated using two examples involving multidisciplinary optimization of a high-speed civil transport model. In these examples, low-fidelity analyses are used to estimate the boundary of the reasonable design space. Within the reasonable design space, response surface models are constructed, based on results from higher-fidelity analyses. The accuracy of the response surface models created with and without using the reasonable design space approach is compared, and the advantages of employing response surface models created via the reasonable design space approach are demonstrated.

I. Introduction

MULTIDISCIPLINARY design optimization (MDO) has become increasingly important as more and more design problems, traditionally solved one discipline at a time, are being analyzed from a multidisciplinary point of view. Numerous methods have emerged in recent years for the modeling of problems involving mutually interacting disciplines. A broad survey of these methods is presented in Ref. 1.

At the Georgia Institute of Technology, a multilevel decomposition approach was applied to the design of a high-speed civil transport (HSCT) wing.[2] In this method, separate system, discipline, and component level analysis tools of varying fidelity were used to estimate a wing weight while considering stress, flutter, and buckling constraints. The HSCT was then sized based on this wing weight. The multilevel decomposition approach was complemented in Ref. 3 by bringing together design and manufacturing considerations early in the design process to reduce time and costs.

The robust design simulation approach was used by Mavris et al.[4] to find economically viable robust designs using the HSCT as a case study. This probabilistic approach views the chosen objective as a probabilistic distribution function determined by input variables that the designer cannot fully control, and seeks a solution that minimizes sensitivity to these variables.

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*Graduate Research Assistant, Multidisciplinary Analysis and Design (MAD) Center for Advanced Vehicles. Student Member AIAA.

†Professor, Department Head, Multidisciplinary Analysis and Design (MAD) Center for Advanced Vehicles. Associate Fellow AIAA.

‡Professor, Multidisciplinary Analysis and Design (MAD) Center for Advanced Vehicles. Associate Fellow AIAA.

§Professor, Multidisciplinary Analysis and Design (MAD) Center for Advanced Vehicles.

¶Professor, Department of Aerospace Engineering, Mechanics, and Engineering Science. Fellow AIAA.

The concurrent subspace optimization approach developed by Sobieszczanski-Sobieski[5] has been used at the University of Notre Dame on a variety of optimization problems.[6,7] In this approach, sensitivity information for coordinating the overall optimization is provided from the global sensitivity equations.[8] One application of this approach is focused on the design-for-manufacturability of integrated circuits,[6] and results in a design tolerant to unavoidable variability in the fabrication process.

The collaborative optimization strategy developed at Stanford University[9,10] is a multidisciplinary design architecture that is well-suited to large-scale multidisciplinary optimization problems. It preserves traditional disciplinary groupings by allowing the parallel execution of disciplinary optimizations. In this method, a system-level optimizer guides the progress of concurrently working discipline-specific optimizers, although convergence to an optimal system design may not occur.

At the Multidisciplinary Analysis and Design (MAD) Center of Virginia Tech, we have applied response surface approximation methods and variable-complexity modeling methods to the MDO of an HSCT aircraft.[11–13] The term *variable-complexity* refers to a design procedure in which refined, computationally expensive analysis techniques are combined with simple, computationally inexpensive techniques.

In many of the approaches to multidisciplinary optimization mentioned earlier, response surface methods (RSM) are used. These techniques construct algebraic approximations, often quadratic polynomials, for the objective function and/or constraints, based on least-square estimates of these functions at a set of sample sites carefully distributed throughout the design space. The optimization or analysis then proceeds on the basis of these approximations.

Other methods for computing the response surface approximations to the data are based on interpolation. They are sometimes referred to as design and analysis of computer experiments (DACE).[14–16] In the DACE approach, the output of the computer code is treated as a realization of a stochastic process and Bayesian analysis is used to estimate correlation coefficients and conditional probabilities.

Response surface modeling methods perform several important functions for MDO. They smooth out the numerical noise often present in some of the response quantities, they ease the integration of codes from various disciplines, they permit disciplinary experts to retain control over their analysis codes rather than turn them over to design optimization generalists, and they are easily adapted to parallel computer architectures.[12–17]

However, RSM becomes increasingly expensive or inaccurate with increasing dimensionality of the design space. This difficulty typically limits RSM to problems with 10–15 design variables. The reasonable design space approach described next is one of the ways to alleviate this difficulty. The reasonable design space approach was successfully applied to HSCT wing configuration optimization[13] and to optimization of the entire HSCT vehicle configuration.[12] Also, structural optimization problems were solved by Roux et al.[18] using this approach.

This paper describes two different applications of the reasonable design space approach to HSCT configuration optimization. In the first application, the response surface approximation is created for structural bending material weight.[12] In the second application, the response surface approximations are created for components of aerodynamic drag.[13]

II. Reasonable Design Space Approach

Most applications of RSM employ a region in a design space that is a box defined in terms of lower and upper bounds on the design variables. However, unless we analyze designs at least at all the vertices of the box, the process of approximation inside the box can entail extrapolation rather than interpolation, with an attendant loss of accuracy. (The words extrapolation and interpolation are used here in their intuitive, rather than technical, senses.) With an n-dimensional box having 2^n vertices, it becomes impractical to evaluate the designs at all of the vertices for values of n of the order of 10 or more. However, other geometrical shapes in n-dimensional space allow interpolation inside with many fewer points. For example, a simplex (the generalization of a triangle and a tetrahedron) has only $n + 1$ vertices. This reflects the fact that a simplex has a much smaller volume than the enclosing box, in fact, smaller by n.

The reasonable design space approach seeks inexpensive constraints that eliminate from consideration large portions of the design box and render it more similar to a simplex or at least an ellipsoid (which also has a much smaller volume than the enclosing box, with a ratio of about 10^{10} for $n = 25$). Such constraints can come from simple geometric constraints that prevent combinations of design variables resulting in unreasonable geometry configurations. However, we also use inexpensive analysis tools to estimate the performance of candidate designs via aerodynamic and performance constraints. These analysis tools may not provide accurate estimates of performance, but they can identify designs with such inferior performance that we can discard them even based on very approximate results.

To define the reasonable design space, we start with an HSCT configuration that lies well inside the design region. This configuration later on will be referred to as the baseline HSCT configuration. A box is constructed around this configuration by permitting each design variable to vary within the allowable bounds. Then one evaluates responses at a number of points in this box selected using some standard statistical design of experiment[19] pattern.

For low dimensional cases a 3^n or 5^n full factorial experimental design, i.e., three or five levels in each variable, is constructed around the baseline HSCT configuration. The HSCT configurations corresponding to these full factorial points are analyzed using the inexpensive, low-fidelity analysis tools and geometric constraints. The results are screened to eliminate from consideration any grossly inferior configurations. The remaining HSCT configurations form the approximation to the reasonable design space.

For high-dimensional cases, a reasonable number of HSCT configurations on the boundary of the box is selected using a standard statistical design of experiment techniques. Inexpensive analysis tools are used at this stage, along with geometric constraints to decide if each particular HSCT configuration is reasonable or not. As the set of the HSCT configurations in this case is sparse (even if we choose 1000 HSCT configurations as our initial sample size, this number is still much smaller than the number of vertices of, say, a 25-dimensional box), the procedure of excluding points to estimate the reasonable design space would eliminate too many HSCT configurations. Instead, the points corresponding to unreasonable HSCT configurations are moved to the edge of the reasonable design space to form an approximation of the boundary of the reasonable design space:

$$x' = \alpha(x - x_b) + x_b, \qquad \alpha > 0 \tag{1}$$

where x and x_b are the points corresponding to unreasonable and baseline HSCT configurations, respectively; and α is a parameter that is adjusted to make the HSCT configuration reasonable. In this process, the points corresponding to unreasonable HSCT configurations are moved toward the center of the design space (baseline HSCT configuration), until the geometric, performance, and aerodynamic constraints are satisfied. A one-dimensional search is performed to determine x'. Note that the HSCT configurations at this stage are analyzed using the inexpensive, low-fidelity analysis tools. Because of that, the cost associated with this search is not prohibitive. In fact, very few complete HSCT analyses are required to complete the search for each point. The details are provided in Sec. V.A. After the points corresponding to unreasonable HSCT configurations have been moved inward, we have a set of points residing on the edge of the reasonable design space. They form an approximation to the boundary of the reasonable design space. This entire process makes sense only if the reasonable design space is starlike with x_b in its kernel [all segments (x_b, x') are feasible]; this appears to be the case for the HSCT problem.

After we have created the approximation to the reasonable design space or to the boundary of the reasonable design space, we can create a response surface model for the reasonable region using more accurate analysis or optimization tools than the ones used to define the reasonable design space. This is done by selecting a subset of the HSCT configurations using, for example, the D-optimality criterion,[20] and then constructing response surface models based on high-fidelity analysis tool results obtained for this smaller subset of HSCT configurations.

III. HSCT Design Problem

In research conducted by our group at Virginia Tech, the design problem is the optimization of an HSCT configuration to minimize takeoff gross weight (TOGW) for a range of 5500 n miles, and a cruise Mach number of 2.4, while carrying 251 passengers. The choice of gross weight as the objective function incorporates structural and aerodynamic considerations, in that the structural considerations directly affect aircraft empty weight and drag, whereas aerodynamic performance dictates the drag, and thus, the required fuel weight. Trim and control requirements are also explicitly treated. For this HSCT design problem, the area-ruled fuselage has a fixed length of 300 ft and an internal volume of 23,720 ft^3. The HSCT configuration and mission are defined using 29 variables (Table 1).

Twenty-five of these variables describe the geometric layout of the HSCT, three variables describe the mission profile, and one design variable refers to engine thrust. Seventy geometric, performance, and aerodynamic constraints are included in the optimization process (Table 2). They are necessary to prevent the optimizer from creating physically meaningless HSCT configurations.

Table 1 HSCT design variables and baseline values[a]

Number	Value	Description
1	181.48	Wing root chord (ft)
2	155.9	LE break point, x (ft)
3	49.2	LE break point, y (ft)
4	181.6	TE break point, x (ft)
5	64.2	TE break point, y (ft)
6	169.5	LE wing tip, x (ft)
7	7.00	Wing tip chord (ft)
8	75.9	Wing semi-span (ft)
9	0.40	Chordwise maximum t/c location
10	3.69	LE radius parameter
11	2.58	Airfoil t/c at root (%)
12	2.16	Airfoil t/c at LE break (%)
13	1.80	Airfoil t/c at tip (%)
14	2.20	Fuselage restraint 1, x (ft)
15	1.06	Fuselage restraint 1, r (ft)
16	12.20	Fuselage restraint 2, x (ft)
17	3.50	Fuselage restraint 2, r (ft)
18	132.46	Fuselage restraint 3, x (ft)
19	5.34	Fuselage restraint 3, r (ft)
20	248.67	Fuselage restraint 4, x (ft)
21	4.67	Fuselage restraint 4, r (ft)
22	26.23	Nacelle 1, y (ft)
23	33.09	Nacelle 2, y (ft)
24	322,617	Mission fuel (lb)
25	64,794	Starting cruise altitude (ft)
26	33.90	Cruise climb rate (ft/min)
27	697.9	Vertical tail area (ft^2)
28	713.0	Horizontal tail area (ft^2)
29	46,000	Maximum sea-level thrust/engine (lb)

[a]LE = leading edge, and TE = trailing edge.

Table 2 Constraints on the HSCT design

Number	Description
1	Fuel volume ≤ 50% wing volume
2	Airfoil section spacing at C_{tip} ≥ 3.0 ft
3–20	Wing chord ≥ 7.0 ft
21	LE break ≤ semispan
22	TE break ≤ semispan
23	Root chord t/c ≥ 1.5%
24	LE break chord t/c ≥ 1.5%
25	Tip chord t/c ≥ 1.5%
26–30	Fuselage restraints
31	Nacelle 1 outboard of fuselage
32	Nacelle 1 inboard of nacelle 2
33	Nacelle 2 inboard of semispan
34	Range ≥5500 n miles
35	C_L at landing ≤ 1
36–53	Section C_l at landing ≤ 2
54	Landing angle of attack ≤ 12 deg
55–58	Engine scrape at landing
59	Wing tip scrape at landing
60	LE break scrape at landing
61	Rudder deflection ≤ 22.5 deg
62	Bank angle at landing ≤ 5 deg
63	Tail deflection at approach ≤ 22.5 deg
64	Takeoff rotation to occur ≤ V_{min}
65	Engine-out limit with vertical tail
66	Balanced field length ≤ 11,000 ft
67–70	Mission segments: thrust available ≥ thrust required

Aerodynamic and performance constraints can only be assessed after a complete analysis of the HSCT configuration is performed; however, the geometric constraints can be evaluated using algebraic relations based on 29 design variables. Thus, we have the opportunity to efficiently identify reasonable designs from the point of view of planform geometry, eliminating nonsensical designs where, for example, the tip chord of the wing is greater than the root chord.

We usually start our optimization from the baseline HSCT configuration. The purpose of the baseline configuration is to provide a point in the interior of the feasible design space. The HSCT baseline geometry configuration was selected based on the previous HSCT optimization studies conducted by our group.[21]

IV. Analysis Methods and Tools

In the conceptual and preliminary phases of the aircraft design, statistically derived experience-based algebraic models, known as the weight functions or weight equations, are often used. For the conceptual level analysis of the aircraft, we use the weight functions from the flight optimization system (FLOPS),[22] and we use algebraic models for the aerodynamic drag analysis. The preliminary level aerodynamic analyses use the vortex lattice method (VLM),[23] the Mach box method,[24,25] and the Harris wave drag code.[26] A simple strip boundary layer is implemented to compute friction drag.[27]

We use two techniques to estimate the stability and control derivatives: empirical algebraic relations from the U.S. Air Force Stability and Control DATCOM,[28] as interpreted by Roskam,[29] and a VLM code developed by Kay et al.[30] The DATCOM methods rely on simple theories and some experimental data, and do not handle unusual aircraft configurations well. The VLM code is better able to handle different configurations, but is more expensive computationally. A more detailed description of these techniques is given in Ref. 31.

Previous studies showed a good correlation between the FLOPS weight function and structural optimization for structural weight prediction.[32] However, work by Huang et al.[32] also indicated that the FLOPS weight function may not be accurate enough in estimating the dependence of the wing bending material weight on planform shape changes for an HSCT-type configuration. Consequently, we have been working on the use of finite element structural optimization to estimate structural bending material weight. The results from the structural optimization are then used in HSCT configuration optimization via RSM. A finite element-based structural optimization code, GENESIS,[33] was used to conduct structural optimization studies of selected HSCT configurations.

The global HSCT configuration optimization was conducted using the penalty function method implemented in the program NEWSUMT-A.[34] We also employed the sequential quadratic programming method as implemented in the optimization code DOT.[35]

V. Wing Bending Material Weight Response Surface Model

To estimate wing bending material weight with acceptable accuracy, our design group used results from the finite element structural optimization. Special mesh and load generators were created to automate the procedure of creating input files for the HSCT structural optimization.

A response surface approximation was created for the wing bending material weight by finding the best least-square quadratic polynomial approximation. We found out that the wing structural weight depends on 25 of our 29 design variables (the leading-edge radius parameter, the starting cruise altitude, the cruise climb rate, and the maximum sea-level thrust per engine were found to not influence wing bending material weight significantly). A quadratic polynomial in 25 variables has 351 coefficients that must be estimated, requiring at least 351 structural optimizations.

A. Identifying the Reasonable Design Space for a Wing Bending Material Weight Response Surface Model

The first step in identifying the reasonable design space was to construct a box, defined by the bounds on the 25 design variables, that encompasses a suitably large region of the design space. Each of the variables, except the fuel weight, was allowed to assume values between 20 and 180% of its baseline value. The fuel weight was only allowed to vary between 75 and 125% of its baseline value because of its strong influence

Table 3 Reduced design space response surface errors[a]

Excluded step	Average error, %	rms error, %	Maximum error, %
Geometrically infeasible designs	13.22	49.49	5532
Geometrically unreasonable designs	0.52	1.01	16.97
Designs with insufficient range	0.35	0.62	11.53

[a]Based on FLOPS weight prediction.

on the mission range, and therefore, the feasibility of the HSCT configuration. A partially balanced incomplete block statistical experimental design was used to select 19,651 HSCT configurations. These were obtained by perturbing one, two, and three variables in such a way that perturbed variables reached their extreme allowable values. This technique is described in detail in Ref. 36. Out of 19,651 HSCT configurations, 83% violated one or more of the HSCT's geometric constraints, and a large portion of the remaining configurations appeared to be unreasonable.

Each point in the design space corresponding to an unreasonable HSCT configuration x was then moved until it resided on the edge of the reasonable design space according to Eq. (1). Computing α in this equation required a set of criteria to determine whether an HSCT configuration was reasonable or not. These criteria were selected carefully to avoid a computationally expensive procedure and to ensure that no reasonable configurations were inadvertently removed.[12] To minimize use of complex constraints, a series of increasingly expensive evaluations were defined and applied in phases (Table 3). Initially, the simple criteria were applied to the data and a large percentage of the candidate points were moved toward a nominal feasible point (HSCT baseline configuration) x_b. These simple constraints, corresponding to the first row in Table 3, mainly prevent nonsensical geometries, such as nacelles not located on the wing or negative airfoil chords. Next, more complex constraints, corresponding to the second row in Table 3, were applied. These constraints did not require complete performance and aerodynamic analysis of the HSCT configuration. Finally, the most complex constraints (third row of Table 3) were applied to complete the relocation of the points. However, as the increasingly complex constraints were applied, fewer of the points had to be moved, and the expense of the constraint evaluations did not become prohibitive.

After each new criterion was applied, a sample response surface was constructed based on the wing bending material weight estimate given by FLOPS weight function. The response surface was created using all 19,651 designs, and the associated modeling error was measured using the same HSCT configurations. As the design space shrank, the accuracy of the response surface approximation improved significantly. Table 3 shows the accuracy of fitting the FLOPS weight function values rather than the results of the structural optimization. However, the improvement in accuracy of the structural optimization results is expected to have a similar trend.

The increase in the accuracy of the response surface model could be explained in part by a significant reduction in the volume covered by the response surface model. In fact, the ratio of the volume of the initial design space box to the volume of the final reasonable design space for our 25-dimensional problem is estimated to be about 10^{14}.

Besides increasing the accuracy of the response surface model, we also eliminate some unreasonable designs (Fig. 1) from our consideration. These designs do not satisfy some geometry and performance constraints. Thus, we eliminate the possibility of performing the costly structural optimization procedure for the HSCT configurations that could not be considered during the global multidisciplinary optimization procedure.

After all 19,651 points in the design space were moved to the edge of the reasonable design space, the D-optimality criterion was used to select two sets of HSCT configurations.

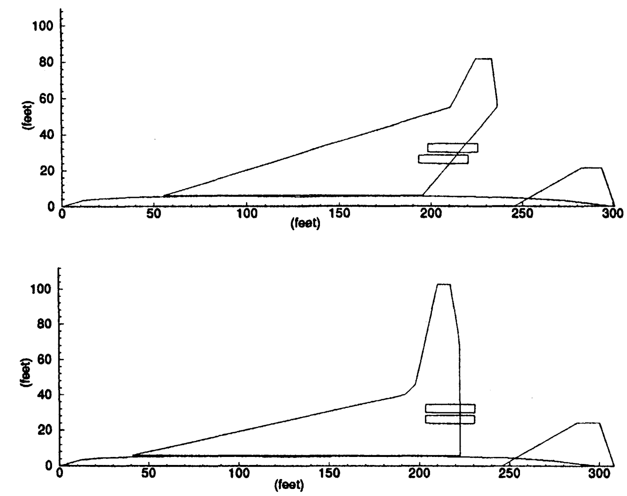

Fig. 1 Examples of the designs that do not satisfy geometric and performance constraints.

Each set contained 1000 configurations. Structural optimization was performed for each HSCT configuration in the two sets. Wing bending material weight was then estimated from the results of the structural optimization. One set of HSCT configurations was used to create a response surface model and the other was used to check the accuracy of this response surface approximation. (In previous studies,[37] it was found that for a six-term polynomial in two dimensions, when the number of configurations to construct a response surface model is at least two times the maximum number of coefficients in a response surface model, the error introduced by a response surface model stabilizes and global trends of the underlying function are accurately approximated.) To obtain the HSCT configurations corresponding to the D-optimal points, we used an implementation of Mitchell's k-exchange algorithm.[38] In Table 4 we compare the accuracies of the FLOPS weight function and the response surface model. Here, the accuracies are calculated based on the differences between FLOPS weight function values and GENESIS structural optimization results, and the response surface model values and GENESIS structural optimization results. However, the results of the structural optimization are noisy, partly as a result of fluctuations in the aerodynamic parameters.[39] For that reason the error shown in the table is an apparent error, which exaggerates the actual values of the response surface model and FLOPS weight function errors. The results indicate that the response surface approximation performs much better than the FLOPS weight function. This is because when we fit the response surface model to a large number of points in the design space, the response surface model filters out the noise and part of the error. Table 4 also reflects the effect of the removal of a large portion of the noise in the aerodynamic parameters, which reduces the error in the structural optimization results used to construct the response surface model.

The response surface model was then used in the overall HSCT multidisciplinary optimization instead of the FLOPS weight function to estimate wing bending material weight. In this manner we incorporated the results from the high-fidelity tool (finite element structural optimization code GENESIS) directly into the overall HSCT optimization process.

Table 4 Comparison of approximations accuracy

Approximation	Average error, %	rms error, %	Maximum error, %
Response surface reduced aerodynamic noise	4.3	6.6	71.1
Response surface with aerodynamic noise	7.7	12.8	114.6
FLOPS	29.8	49.1	297.3

Table 5 Comparison of HSCT optimal designs

Parameter	FLOPS	Response surface
Planform geometry		
Root chord (ft)	161.5	158.3
Tip chord (ft)	7.44	7.65
Wing semispan (ft)	60.9	60.6
Aspect ratio	1.74	1.79
Wing area (ft^2)	10,263	9927
Performance data		
Range (n mile)	5510	5505
Landing angle of attack (deg)	11.87	12.01
L/D_{max}	8.98	8.91
Weight data		
W_g (lb)	622,551	618,168
W_b (lb)	18,755	18,224
W_{b_G} (lb)	22,848	21,517
W_f (lb)	331,821	330,248

As an alternative to the use of response surfaces, the structural optimization program (GENESIS) could have been integrated into the configuration optimization. With derivatives calculated by finite differences, this would have required a comparable number of structural optimizations, but would have had the following disadvantages:

1) The optimum structural design is not a smooth function of the configuration shape, and incomplete convergence of the optimization adds numerical noise. This nonsmoothness would hamper the use of a derivative-based optimizer.

2) Code integration would require a substantial effort as GENESIS is a large and complex program.

3) A small number of structural optimizations did not converge properly and were discarded from the response surface. When such problems happen during the process of optimization they can go undetected and thwart the progress of the optimizer.

B. Using Wing Bending Material Weight Response Surface Model in HSCT Design Optimization

Complete HSCT design optimizations were performed to evaluate the effect of using the response surface approximation for wing bending material weight. At the completion of the optimizations, results were compared with structural optimization results. The response surface model was used by modifying the takeoff gross weight calculations within the weight module of FLOPS. In place of FLOPS weight function estimates for wing bending material weight, the response surface model predictions were used. However, the response surface model is intended for use only in the reasonable design space, and its predictions could not be relied upon outside this region. Therefore, all calculations outside the reasonable design space were done with the original FLOPS weight function. At the edge of the reasonable design space, a smoothing function was used to prevent a discontinuity between the wing bending material weight calculated by FLOPS and the response surface model predictions.[12] Results from the optimizations are given in Table 5 and the planforms are plotted in Fig. 2.

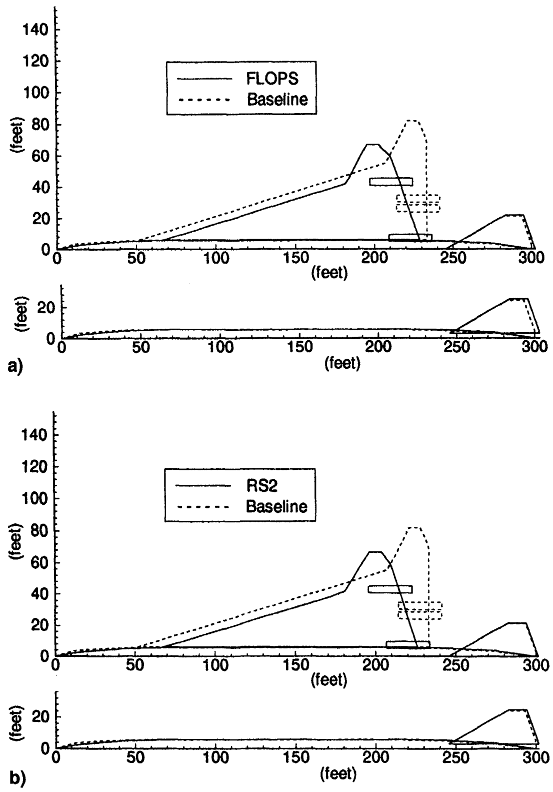

Fig. 2 Optimal HSCT planforms obtained using a) FLOPS and b) response surface compared with baseline configuration.

In Table 5, W_g corresponds to the takeoff gross weight of the aircraft, W_b is the wing bending material weight estimated by either the FLOPS weight function or the response surface model, W_{b_G} is the wing bending material weight obtained using GENESIS structural optimization results, and W_f denotes the weight of required fuel. Comparing W_b for the case when the FLOPS weight function was used to estimate wing bending material weight and W_{b_G}, one can see the difference of about 4100 lb, or 17.9% of W_{b_G}. For the optimization case when the response surface model was used to estimate wing bending material weight, this difference is about 3300 lb, or 15.3% of W_{b_G}. Thus, the results provided by the response surface approximation are slightly more accurate than the results from the FLOPS weight function. This leads to savings in the takeoff gross weight of about 4400 lb between the two optimal HSCT configurations.

Results of applying response surface approximation for the wing bending material weight indicate that the use of the response surface model led to an improvement in performance compared with the use of the FLOPS weight equation.

VI. Aerodynamic Response Surface Models

As a second example, we have constructed response surface approximations to four aerodynamic quantities: volumetric wave drag ($C_{D\text{wave}}$), two components of supersonic drag-due-to-lift [leading-edge suction (C_T/C_L^2) and lift curve slope ($C_{L\alpha}$ at $M = 2.4$)], and the subsonic lift curve slope ($C_{L\alpha}$ at $M = 0.2$).

As the first step, we have limited our attention to 10 design variables that describe the geometry of the wing. We also used a smaller box in the design space, corresponding to only ±20% changes in design variables.

The 10 variables for this design problem are wing root chord (C_{root}), wing tip chord (C_{tip}), wing semispan (b_{half}), inboard leading-edge sweep angle (Λ_{LE_I}), outboard leading-edge sweep angle (Λ_{LE_O}), chordwise location of maximum thickness ($t_{\max}$), leading-edge radius parameter (R_{LE}), thickness-to-chord ratio (t/c), spanwise location of the inboard nacelle (b_{nacelle}), and fuel weight (W_{fuel}).

A. Identifying the Reasonable Design Space for Aerodynamic Response Surface Models

A 3^{10} full factorial experimental design, i.e., three levels in each variable, was constructed around the baseline HSCT configuration. The 59,049 (3^{10}) HSCT configurations corresponding to the full factorial design points were analyzed using the inexpensive, conceptual level analysis tools, and the HSCT analysis results were screened to eliminate from consideration any grossly infeasible HSCT configurations. Here, an HSCT configuration was eliminated if it violated any of the geometric constraints by more than 5%. After the screening process, only 29,163 HSCT configurations remained, with 51% of the original HSCT candidate configurations having been eliminated. One would expect the response surface approximations for the reasonable design space to be more accurate than response surface approximations for the entire design space, as they were in the case of the wing bending material weight response surface model.

To investigate this question of response surface model accuracy, 132 HSCT configurations were selected based on the D-optimality criterion from the set of 3^{10} HSCT configurations defined by the full-factorial experimental design in the original design space. These 132 HSCT configurations were evaluated using the inexpensive, conceptual-level analysis tools and full quadratic response surface models were constructed for the four aerodynamic quantities previously described. These response surface models are valid over the entire original design space. Similarly, response surface models were constructed for 132 HSCT configurations corresponding to D-optimal points chosen from 29,163 reasonable configurations.

Next, 150 HSCT configurations were randomly selected from the 29,163 HSCT configurations in the reasonable design space and were evaluated using inexpensive analysis tools. From this exact data for the randomly selected HSCT configurations, the average, rms, and maximum errors were calculated for each of the four aerodynamic quantities using the response surface models obtained for the original design space and for the reasonable design space (Table 6).

As shown in Table 6, the response surface models for the reasonable design space are more accurate than the RS models for the original design space for C_T/C_L^2, $C_{L\alpha}$ at $M = 2.4$, and $C_{L\alpha}$ at $M = 0.2$. Somewhat surprisingly, the error calculations indicate that accuracy of the response surface model for $C_{D\text{wave}}$ is slightly lower for the reasonable design space as compared to the original design space. Further investigations showed that errors in the response surface model for $C_{D\text{wave}}$ were particularly sensitive to the location in the design space of the 150 randomly selected HSCT configurations. Thus, the errors calculated for $C_{D\text{wave}}$ using both sets of response surface models are not significantly different. Although the improvements in accuracy shown here for three of the four response surface models are not quite as substantial as those for the wing bending material weight response surface models, this investigation again demonstrates that an improvement in accuracy is obtained by limiting the response surface model construction to the reasonable design space.

Table 6 Response surface modeling error estimates based on analyses of 150 randomly selected HSCT configurations

Parameter	Average error, %	rms error, %	Maximum error, %
$C_{D\text{wave}}{}^{a}$	6.89	8.33	30.85
$C_{D\text{wave}}{}^{b}$	7.10	9.40	40.91
$C_{L\alpha}$ at $M = 2.4^{a}$	0.66	0.82	2.18
$C_{L\alpha}$ at $M = 2.4^{b}$	0.40	0.58	1.82
C_T/C_L^{2a}	28.70	47.28	157.57
C_T/C_L^{2b}	12.03	18.93	73.59
$C_{L\alpha}$ at $M = 0.2^{a}$	1.42	1.81	4.59
$C_{L\alpha}$ at $M = 0.2^{a}$	0.45	0.56	1.49

[a]Response surface model, original design space. [b]Response surface model, reasonable design space.

B. Using Aerodynamic Response Surface Models in HSCT Design Optimization

After the aerodynamic response surface models were created, they were used in the HSCT configuration optimization studies. With the use of aerodynamic response surface models, the optimizer converged to virtually identical configurations from different starting points. That was not the case when configuration optimization was performed without the use of the response surface models. This is consistent with the convergence difficulties and occurrence of multiple very localized optima, such as those generated by numerical noise, which originally motivated the use of response surface modeling methods. One such locally optimal HSCT configuration has the takeoff gross weight 17,000 lb heavier than the HSCT configuration obtained with the help of the aerodynamic response surface models.

Figure 3 shows the wing planform changes from the initial to the optimal HSCT configuration when response surface models were used in the optimization procedure. The locally optimal HSCT configuration obtained without using response surface models is also shown in Figure 3. The initial and optimal values of design variables for these configurations are listed in Table 7.

The optimal HSCT configuration obtained using the response surface models has a smaller wing area and a lower aspect ratio than the initial HSCT, which results in a wing

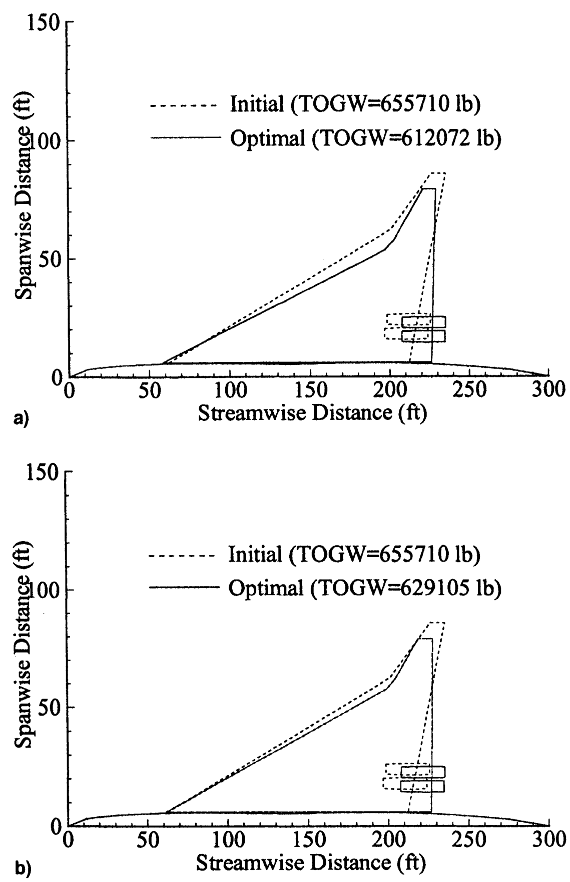

Fig. 3 Ten variable initial and optimal wing obtained a) using and b) without using response surface models.

Table 7 Initial and optimal parameters for the 10 variables HSCT optimization problem

Variable	Initial	Optimal with response surface	Optimal without response surface
C_{root}	150.0 ft	167.7 ft	165.3 ft
C_{tip}	9.0 ft	8.1 ft	9.1 ft
b_{half}	80.0 ft	73.4 ft	73.1 ft
Λ_{LE_I}	68.0 deg	71.1 deg	69.5 deg
Λ_{LE_O}	47.0 deg	40.3 deg	40.3 deg
t_{max}	33.0%	47.5%	36.2%
R_{LE}	2.5	3.2	2.5
t/c	2.1%	2.3%	2.0%
$b_{nacelle}$	18.0 ft	16.7 ft	16.7 ft
W_{fuel}	319,000 lb	300,700 lb	310,550 lb
TOGW	655,710 lb	612,072 lb	629,350 lb

structural weight savings of 23,500 lb, a decrease of 18.3%. In addition to the structural improvements, the $(L/D)_{max}$ of the HSCT increased from 9.37 to 9.54, an improvement of 1.8%.

VII. Concluding Remarks

For a variety of reasons, including attempts to reduce numerical noise and facilitate software integration, response surface models are increasingly used in MDO. However, as the number of design variables increases, the accuracy of the response surface approximations deteriorates and their computational cost grows. This paper describes what we have termed a reasonable design space approach that alleviates some of these problems. The reasonable design space approach employs geometric constraints and constraints based on simple, computationally inexpensive analytical tools to eliminate large portions of the design space from consideration. This reduction in the size of the design space substantially improves the accuracy of the response surface models.

In this paper, the reasonable design space approach is demonstrated through two applications to HSCT configuration optimization. In the first application, a response surface model is used to estimate the structural bending material weight. In the second application, response surface models are used to estimate the aerodynamic lift and drag coefficients. Both applications use quadratic polynomial response surface models.

For the case of the structural bending material weight, the reasonable design space approach reduces the size of the design space significantly, and results in large improvements in the accuracy of the response surface model. Furthermore, the response surface approximation provides a more accurate representation of the bending material weight than that obtained from a general-purpose weight equation. For the case of aerodynamic response quantities, the reasonable design space approach eliminates about half of the design space, and results in more modest improvement in the accuracy of the response surface models.

It is also demonstrated that the use of response surface models can eliminate the problem of very localized optima, such as those generated by numerical noise, which plagued the numerical HSCT configuration optimization procedure when the original noisy analysis methods were used.

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